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I. **Solution by C. N. SCHMALL, New York City.**

Construction: Let P be the given point and ABC the given triangle. Join D, E, F, the middle points of the sides. Now draw PGI parallel to BC and join EG. From D draw DH parallel to GE, and then draw HI parallel to BA and meeting PG prolonged in I. On PI as a diameter describe a semi-circle and thereon lay off $PK=PG$. Draw KI, and in the base BC lay of $HM=KI$. Draw PM cutting AB in L. The line PLM bisects the triangle and is the line required.

Proof: Let PM and HI meet in N. Now the triangles PGL, PIN, MHN are clearly similar.

Also, since $PI^2=PK^2+KI^2=PG^2+HM^2$ (for $PK=PG$, and $KI=HM$).

$\therefore \triangle PIN=\triangle PGL+\triangle MHN$...(1).

Hence, the quadrilateral $LGIN$ is equal to the triangle MHN;

$\therefore$ triangle BML=parallelogram $BHIG$...(2).

Again, GE and DH are parallel;

$\therefore GB : BE = DB : BH$...(3).

Hence, the parallelograms $BHIG$ and $BEFD$ have a common angle B, and the including sides are reciprocally proportional.

$\therefore$ by (3), parallelogram $BHIG$=quadrilateral $BEFD$.

But by (2), parallelogram $BHIG$=triangle BML...(2).

$\therefore$ triangle BML=parallelogram $BEFD=\frac{1}{2}$ triangle ABC.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

Let P be the given point. Draw PE parallel to AB, cutting AC in D; make parallelogram $DEFA=\frac{1}{2}$ triangle ABC. On AB at F erect perpendicular $FG=PD$, and make $GA=PE$. Connect P with Q, then PQ will be the required line which bisects triangle ABC.

For, $\triangle FHQ : \triangle PHE=FQ^2 : PE^2=PE^2-PD^2 : PE^2$.

$$\therefore \frac{\triangle FHQ}{\triangle PHE}=1-\frac{PD^2}{PE^2}=1-\frac{\triangle PDH}{\triangle PEH}.$$

$\therefore \triangle FHQ=\triangle PEH-\triangle PDI=DIHE$; $\therefore FHQ+IHFQ=DIHE+IHFA=DEFA$.

Note: To construct $DEFA$, connect C with the mid-point M of AB, draw DM, and CN parallel to DM, then F will be the mid-point of AN.

Also solved by G. B. M. Zerr, V. M. Spunar, and Daniel B. Northrup.

NOTE. The following gentlemen should have received credit for solving 342: J. A. Caparo, V. M. Spunar, and S. Lefsehetz.

CALCULUS.

274. **Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.**

About the vertices of a regular tetrahedron four spheres are drawn with radii equal to the edge of the tetrahedron. Find the volume common to them all.

Solution without the use of the Calculus by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

One vertex of the tetrahedron is at the center of each sphere while the other three vertices are in the surface.

Hence the total volume common to all the spheres=the volume of the tetrahedron+the volumes of four equal spherical segments.

Let a=the edge of the tetrahedron and also the radius of each sphere. Then $\frac{1}{3}a\sqrt{6}$=altitude of tetrahedron, and $\frac{1}{12}a^3\sqrt{2}$=its volume.

$a-\frac{1}{3}a\sqrt{6}=\frac{a}{3}(3-\sqrt{6}$=altitude of segment. $\frac{\pi a^3}{81}(3-\sqrt{6})^2(6+\sqrt{6})=$ $\frac{\pi a^3}{27}(18-7\sqrt{6})$=volume of spherical segment.

Total volume required $=V=\frac{4\pi a^3}{27}(18-7\sqrt{6})+\frac{a^3\sqrt{2}}{12}$.

$\therefore V\frac{a^3}{108}(9\sqrt{2}+288\pi-112\pi\sqrt{6})$.

This solid might be called a spherical tetrahedron.

275. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

Explain fully why the circular measure of an angle is used in the calculus.

No satisfactory answer of this problem has been received.

276. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In a certain country the tax per $1 on a person's income varies as the cube root of the number of dollars, and when the income is $8000 the rate per dollar is 5 cents. Find the largest net income possible.

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Let x=the number of dollars, gross income; y=the rate of taxation on $1; xy=the tax on x dollars; and u=the net income; from which we have $u=x-xy$...(1).

From the problem we have, $(8000)^{\frac{1}{3}} : x^{\frac{1}{3}} :: 0.05 : y$; or $y=x^{\frac{1}{3}} / 400$...(2); and from (2), $xy=x^{\frac{4}{3}} / 400$...(3). Substitute in (1) the value of xy from (3), and we have, $u=x-x^{\frac{4}{3}} / 400$...(4). Differentiate (4) and we have, $du/dx=1-x^{\frac{1}{3}} / 300=0$; or x=$27,000,000. Substitute this value of x in (2), and we have, $y=\frac{3}{4}$, the rate of taxation. Therefore the net income is $\frac{1}{4}$; or $\frac{1}{4}$ of $27,000,000=$6,750,000, the maximum net income.

To find when the rate of taxation is prohibitive, let z=the gross income, and we have, $(8000)^{\frac{1}{3}} : z^{\frac{1}{3}} :: 5\% : 100\%$; or z=$64,000,000. That is to say, when the gross income is $64,000,000, the whole income is taken for the tax.

Also solved by H. C. Feemster, J. Scheffer, J. E. Sanders, V. M. Spunar, S. A. Corey, J. W. Clawson, and the Proposer.